

Creating Naked Singularities and Negative Energy¹

Gary T. Horowitz

Department of Physics, UCSB, Santa Barbara, CA 93106
gary@physics.ucsb.edu

Abstract

A brief review is given of three recent results concerning classical solutions of gravitational theories: (1) With asymptotically anti de Sitter boundary conditions, there are matter theories satisfying the positive energy theorem which violate cosmic censorship. (2) Despite supersymmetry, there are solutions to $\mathcal{N} = 8$ supergravity in which the total gravitational energy is arbitrarily negative. This theory can also violate cosmic censorship. (3) A large class of supersymmetric compactifications (including all simply connected Calabi-Yau manifolds) have solutions with negative four dimensional effective energy density.

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1 Introduction

I would like to review some surprising results concerning classical solutions of gravitational theories. Since many of these theories arise as the low energy limit of string theory, these results have direct implications for string theory and suggest a new approach for studying cosmological singularities in a quantum theory of gravity. I will only have time to describe the main ideas and summarize the results. This work was all done in collaboration with T. Hertog and K. Maeda, and the reader is referred to the original papers for more details [1, 2, 3].

Between 1965 and 1970, Hawking and Penrose proved a series of powerful theorems showing that large classes of solutions to Einstein's equation are singular [4]. These singularity theorems marked a major advance in our understanding of strong gravitational fields. However, they say nothing about event horizons. In particular, they do not prove that black holes must form in gravitational collapse. It is possible that, in some cases, one forms singularities that are visible to distant observers. These are known as *naked singularities*. In 1969, Penrose suggested that there might be a “cosmic censor” that forbid naked singularities from forming [5]. This has become known as cosmic censorship. I should emphasize that cosmic censorship is not concerned with static timelike singularities like the one in the negative mass Schwarzschild solution. Those singularities are indeed naked, but they are present for all time. They are part of the initial conditions. Cosmic censorship deals with singularities arising from nonsingular initial data.

Cosmic censorship is clearly important for our understanding of black holes. In fact, our entire theory of black holes is based on the assumption that there are no naked singularities outside the event horizon. But despite extensive work over the past three decades, we are still very far from a complete proof [6]. When something is hard to prove, it is often fruitful to look for counter-examples, and many people have done so. It was shown early on that if you model matter by pressureless dust, then it is easy to produce naked singularities [7]. One can take a spherical ball of dust and start the outer shells collapsing inward faster than the inner shells. When the shells cross, the density diverges and one gets a curvature singularity which can lie outside the event horizon. But this is clearly an artifact of the unphysical model of matter. Real matter has pressure. A clear signal of the unphysical nature of this

example is that if one turns off gravity and just studies dust in flat space, one has the same type of singularities. Since we are interested in singularities arising from gravitational collapse, we should only consider matter which is nonsingular when evolved in Minkowski space, such as a scalar field.

About ten years ago, Choptuik showed that spherically symmetric scalar fields coupled to gravity can produce a naked singularity [8]. This attracted a lot of attention (and caused Stephen Hawking to lose a bet with Kip Thorne) but it did not really disprove cosmic censorship. The reason is that Choptuik had to fine tune his initial data. Nearby initial data either forms small black holes or results in the scalar field scattering and never forming a singularity. Cosmic censorship deals with generic initial data. To violate it, one needs to have an open set of initial data which evolve to form naked singularities.

The above examples assume the usual boundary conditions for an isolated system: the spacetime is asymptotically flat at infinity. However there has recently been a great deal of interest in spacetimes with nonzero cosmological constant. When the cosmological constant is positive (as suggested by the astrophysical data) and the matter Lagrangian includes dilatons (as suggested by string theory) there is an interesting class of counterexamples to cosmic censorship which has received less attention [9]. The action can be taken to be

$$S = \int [R - 2(\nabla\phi)^2 - e^{-2\phi}F^2 - 2\Lambda + \mathcal{L}_m] \sqrt{-g} \, d^4x \quad (1.1)$$

where F is a Maxwell field, $\Lambda > 0$, and \mathcal{L}_m describes some charged matter. Cosmic censorship can be violated since there are no static charged black hole solutions in this theory [10]. Roughly speaking the reason for this is that a static black hole in de Sitter space is expected to have at least two horizons; a cosmological horizon as well as a black hole horizon. On a static surface, the scalar field ϕ should reach both a local maximum and a local minimum. But its field equation $\nabla^2\phi = -\frac{1}{2}e^{-2\phi}F^2$ only allows ϕ to have maximum or minimum values (but not both) whenever $F^2 \neq 0$. One can construct data in which the charged matter collapses and forms singularities, but since there are no static black holes, the singularities must be naked.

I will focus on the case of negative cosmological constant. I realize that this is the wrong sign as far as cosmologists are concerned, but in terms of understanding string theory, we are in much better shape for several reasons. Most importantly, we have

a complete nonperturbative formulation of the theory provided by the AdS/CFT correspondence [11]. AdS refers to anti de Sitter space, the maximally symmetric solution with negative cosmological constant, and CFT corresponds to a conformally invariant quantum field theory. The remarkable claim (which is supported by a large body of evidence [12]) is that string theory with asymptotically AdS boundary conditions is completely equivalent to a CFT.

It turns out that cosmic censorship is much easier to violate in asymptotically AdS spacetimes than asymptotically flat spacetimes. This is a result of two facts:

- 1) Black holes are harder to form.
- 2) Singularities are easier to form.

The first just follows from the form of the solution for a black hole in AdS

$$ds^2 = - \left(1 - \frac{2M}{r} + \frac{r^2}{\ell^2} \right) dt^2 + \left(1 - \frac{2M}{r} + \frac{r^2}{\ell^2} \right)^{-1} dr^2 + r^2 d\Omega \quad (1.2)$$

where ℓ is the radius of curvature of AdS and M is the mass of the black hole. (If $M = 0$, the metric is just AdS.) This shows that the mass needed to form a black hole of size $R_s \gg \ell$ is $M \propto R_s^3$. This is much larger than the mass needed to form the same size black hole in asymptotically flat space which is $M = R_s/2$.

The second fact is a result of a qualitatively new way to form singularities with a scalar field in AdS. In asymptotically flat spacetimes, the main way to form a singularity is to arrange the scalar field so that there is a large energy density inside a small volume. This will produce a singularity, but it also will usually produce a black hole. Now consider a potential $V(\phi)$ for a scalar field with a negative minimum. Suppose we take initial data in which the scalar field is simply a constant ϕ_0 (with $V(\phi_0) < 0$) and $\dot{\phi} = 0$. By homogeneity, when this data is evolved, the metric can be written in Robertson-Walker form

$$ds^2 = -dt^2 + a^2(t) d\sigma_{-1} \quad (1.3)$$

where $d\sigma_{-1}$ denotes the metric on a unit hyperboloid. If ϕ_0 is at the minimum of the potential, it will stay there for all time and the solution for the scale factor is $a(t) = \ell \cos(t/\ell)$ with $\ell^2 = -3/V(\phi_0)$. This is just AdS in different coordinates than (1.2). In particular, $a(t) = 0$ is just a coordinate singularity. But if ϕ_0 is even slightly away from the minimum, the solution is dramatically changed. Under evolution, the

scalar field will start to oscillate. Einstein's equation still forces the scale factor to vanish, but now when it vanishes, the energy in the scalar field becomes infinitely blue-shifted and one has a curvature singularity.

2 Cosmic censorship violation in AdS

We will use the above two facts to show that there are theories of gravity coupled to a scalar field ϕ which violate cosmic censorship [1]. Our strategy will be to find a potential $V(\phi)$ such that there is an open set of initial data which forms a singularity in a large central region, but such that there is not enough mass to produce a black hole big enough to enclose the singularity. For now we will not insist that V comes from string theory, but ask if there is any potential which will violate cosmic censorship. One physical condition that we will impose is that V satisfies a positive energy theorem. Recall that the total energy is well defined for every asymptotically AdS spacetime [13]. The positive energy theorem states that the total energy of all nonsingular initial data is positive and vanishes if and only if the metric is AdS everywhere [14, 15, 16]. If this fails, the asymptotic AdS space is likely to be unstable, and the theory may not have a ground state.

We now show that there indeed exist $V(\phi)$ such that the positive energy theorem holds but cosmic censorship is violated. The idea is to consider a potential like that shown in Fig. 1. $V(\phi)$ has a global minimum at $\phi = 0$ and a local minimum at $\phi = \phi_1$, both of which are negative. We will require that $\phi \rightarrow \phi_1$ asymptotically. One might worry that the positive energy theorem would be violated since if one keeps ϕ very small inside a large ball and then sends it over the barrier to ϕ_1 in a thin shell, it would appear that the total energy is less than if $\phi = \phi_1$ everywhere. However this intuition fails because the negative energy density causes the space to be negatively curved, and a negatively curved hyperboloid has the property that there is as much volume inside a shell at large radius as there is inside the entire ball of the same radius. When one computes the total energy, it turns out that the positive energy theorem typically is satisfied if the barrier is large enough, but is violated if the barrier is too small. Since we want to keep the total mass small, we will adjust the height of the barrier to be close to the transition point, but still satisfy the positive energy theorem.

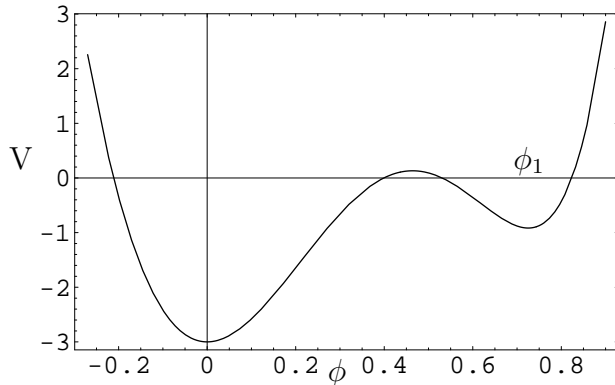


Figure 1: A potential $V(\phi)$ that satisfies the positive energy theorem for solutions that asymptotically approach the local (AdS) minimum at ϕ_1 , but which violates cosmic censorship.

Now consider spherically symmetric initial data with $\phi(r) = \epsilon$ for $r < R_0$, $\phi = \phi_1$ for $r > R_1$ and continuously interpolating in between. Here $R_1 > R_0$ are two large radii. We also assume that $\dot{\phi} = 0$ so the initial data is time symmetric. By spherical symmetry, we can assume the metric takes the form

$$ds^2 = \left(1 - \frac{2m(r)}{r} + \frac{r^2}{\ell^2}\right)^{-1} dr^2 + r^2 d\Omega. \quad (2.1)$$

where now $\ell^2 = -3/V(\phi_1)$. The mass function $m(r)$ is uniquely determined by the Einstein scalar constraint for any $\phi(r)$. The total mass is just the limit of $m(r)$ as $r \rightarrow \infty$. Under evolution we know that the region $r < R_0$ will become singular in a time of order $V(0)^{-1/2}$. The question is whether this can be hidden inside a black hole. If a black hole eventually forms we can trace the null geodesic generators back to our initial surface where they will form a sphere of radius R_s (see Fig. 2). The black hole area theorem still holds even with $V < 0$, since it only requires the null convergence condition $T_{\mu\nu}k^\mu k^\nu \geq 0$ for all null k^μ . So $R_{bh} \geq R_s$. We can now ask if there is enough mass to form a black hole of size R_s .

This is just a question about our initial data and is easy to answer. It turns out that for some $\phi(r)$ the answer is no! In fact, it is not even close. Recall that a large Schwarzschild-AdS black hole has a mass $M \propto R_s^3$. This theory also has black holes with scalar hair, but the positive energy theorem ensures that a nontrivial scalar field outside the horizon only increases the mass. In the extreme case where we adjust

the barrier to be right on the verge of violating the positive energy theorem (but still satisfying it) and choose $\phi(r)$ to minimize the mass, one finds that the mass available is $M \propto R_1$, but the mass needed to form a black hole is $M_{bh} \propto R_1^3$ (since in this case, $R_s \propto R_1$). Because of this large discrepancy, small changes in the initial data will not effect this conclusion. One can take nearby initial data which is not spherical or time symmetric and still not have enough mass to form a black hole large enough to enclose the singularity. The net result is that one has an open set of initial data which produce naked singularities and cosmic censorship is violated in this theory.

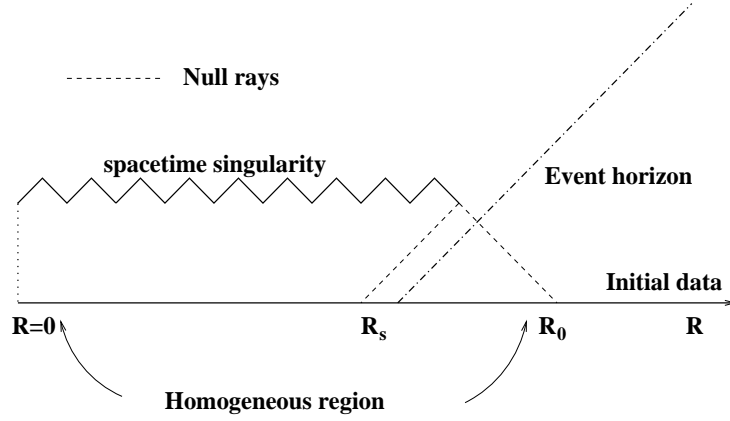


Figure 2: *If an event horizon encloses the singularity, it must have an initial size greater than R_s .*

Since we have gravitational collapse to a singularity which cannot be enclosed inside a static black hole, we clearly have a violation of the standard black hole paradigm. But is there really a naked singularity? Inside the homogeneous region, the singularity is spacelike and cannot be called naked. However, we expect the singularity to extend outside the region of homogeneous evolution. We don't know the exact solution here but there are only a few possibilities. The most likely outcome is that the singularity simply ends, in which case the endpoint is naked. This includes the possibility that the singularity becomes timelike, because at that point the classical evolution stops. (There is no unique evolution when the singularity is timelike since we don't know what boundary conditions to impose there.) Another possibility is that the singularity remains spacelike and extends to infinity. This is a disaster much worse than a naked singularity, since then localized initial data with finite energy has created a big crunch singularity which cuts off spacetime everywhere. We have

not ruled this out, but find it very unlikely. A final possibility is that the singularity could become null. This is similar to a naked singularity in that regions of arbitrarily large curvature are visible to distant observers. Which of these possibilities actually occurs can be settled using numerical relativity. This is currently being investigated [17].

3 Negative energy and naked singularities in supergravity

We now try a similar construction with a potential coming from string theory. The low energy limit of string theory with $AdS_5 \times S^5$ boundary conditions is $D = 5$, $\mathcal{N} = 8$ supergravity. This theory has scalars with $m^2 < 0$. More than twenty years ago, Breitenlohner and Freedman [18, 19] showed that these tachyons will not cause an instability in AdS_d provided $m^2 \geq -(d-1)^2/4$ (where we have set the AdS radius to one). This is known as the Breitenlohner-Freedman (BF) bound. Supergravity has fields which saturate this bound. Since our goal is to construct solutions with low total energy, it is natural to use these fields. The relevant potential $V(\phi)$ now has a negative maximum at $\phi = 0$ and falls off exponentially for large $|\phi|$. There are no local minima and no potential barrier.

We want to consider initial data with $\phi = \phi_0$ for $r < R_0$ and then $\phi \rightarrow 0$ asymptotically. The question is at what rate should ϕ go to zero. We clearly want ϕ to go to zero as slowly as possible. This will minimize the positive contribution to the energy from the spatial gradients and maximize the negative contribution to the energy coming from the fact that the potential is less than its asymptotic value over a larger region. It turns out that the slowest ϕ can fall off and keep the total mass finite is $1/r^2$. So we take

$$\phi(r) = \frac{A}{R_0^2} \text{ for } r < R_0, \quad \phi(r) = \frac{A}{r^2} \text{ for } r > R_0. \quad (3.1)$$

Assuming zero time derivatives, one can now solve the constraint for the spatial metric and compute the total mass. Surprisingly, it turns out that it is negative [2], $M \propto -A^2$! Furthermore, by increasing A , we can make the total mass arbitrarily negative. There is no lower bound.

How can this be? The theory is supersymmetric, and AdS is a supersymmetric solution. There is supposed to be a positive energy theorem [14, 15, 16] which ensures that this cannot happen. Furthermore, the AdS/CFT correspondence says that this theory is supposed to be equivalent to the CFT which has a Hamiltonian bounded from below.

To resolve this contradiction, let us review the positive energy theorems. The original Breitenlohner-Freedman argument applied to test fields satisfying the Klein-Gordon equation in AdS. If ξ^μ denotes the global timelike Killing field, the energy is just the integral of $T_{\mu\nu}\xi^\mu$ over a spacelike surface. This integral is not positive definite since $m^2 = -4$. But if one sets $\phi = \psi/(1+r^2)$ one can integrate by parts and rewrite this energy as

$$E = \frac{1}{2} \int \left[(\dot{\psi})^2 + (1+r^2)(D\psi)^2 + 4\psi^2 \right] \frac{r^3}{(1+r^2)^3} dr d\Omega_3 - \oint_\infty \psi^2 d\Omega_3 \quad (3.2)$$

The surface term vanishes if ϕ falls off faster than $1/r^2$, and in this case the energy is manifestly positive. For all fields with m^2 above the BF bound, finiteness of the energy requires the field to fall off faster than $1/r^2$. However, for fields which saturate the bound, we have seen that $1/r^2$ fall off is allowed by finite energy. The surface term is now nonzero and negative causing $E < 0$. Breitenlohner and Freedman suggested that one should use an “improved” $T_{\mu\nu}$ which corresponds to adding a $\beta R \phi^2$ term to the Lagrangian. For a test field, this does not change the equation of motion for ϕ since R is a constant in AdS and acts like a mass term. But in the full theory, this changes the gravitational dynamics. One can check that $\mathcal{N} = 8$ supergravity does not include these terms. So one cannot use $\beta \neq 0$ to make the energy positive.

There is a complete nonlinear proof of the positive energy theorem in AdS which is a generalization of the spinorial proof of positive energy in asymptotically flat spacetimes originally found by Witten [20]. One solves a Dirac like equation for a spinor on a spacelike surface $\gamma^i \hat{\nabla}_i \epsilon = 0$ where $\hat{\nabla}_i$ is a supercovariant derivative (the derivative which appears in the supersymmetry transformation laws). One can then derive an identity in which a surface term is equal to a manifestly positive volume integral. If ϕ falls off faster than $1/r^2$, the surface term is the usual total energy in AdS and one has a positive energy theorem. However, if $\phi \sim 1/r^2$ asymptotically, there is another contribution to the surface term at infinity coming from the asymptotic scalar field. (In a test field limit, this extra contribution cancels the negative surface

term in (3.2).) The sum of the two surface terms must be positive, but the usual energy need not be.

The net result is that the positive energy theorem requires boundary conditions which are stronger than finite mass, and $M < 0$ solutions exist. However there is a modified energy (corresponding to the entire surface term in the nonlinear proof described above) which is always positive. This implies that AdS is stable: It cannot decay to another zero energy solution. It also suggests that the CFT Hamiltonian should be identified with this modified energy. This can probably be verified by computing the supercharge, or more generally, all the charges associated with the asymptotic superalgebra along the lines of [21]. Why not call this modified energy the “real energy” and forget the original definition? The answer is that the usual energy still governs the asymptotic behavior of the metric. Test particles in the bulk solution at large radius will feel a negative gravitational mass.

Having clarified the negative energy, we return to the question of evolution of our initial data (3.1). The central region collapses to a singularity. If the total energy was conserved, then this singularity could not be hidden inside a black hole since the total energy is negative and black holes must have positive mass. However it turns out that for fields that saturate the BF bound and fall off like A/r^2 (and only in this case), the energy is not automatically conserved. There can be a nonzero flux of energy through infinity if the coefficient A becomes time dependent. If the energy grows sufficiently, a black hole could form.

To control this, we can impose a large radius cut-off R_1 and require that $\phi(R_1) = A/R_1^2$ is fixed. This is automatically implemented in most numerical evolution schemes, and is a standard regulator in discussions of AdS/CFT. However a subtlety now arises which can be seen as follows. In the asymptotic AdS region, the modes of a scalar field with $m^2 = -4$ either fall off like $1/r^2$ or $\ln r/r^2$. In the absence of a cut-off, only the $1/r^2$ modes have finite energy and the $\ln r/r^2$ modes play no role. However with a cut-off, modes that behave like $\ln r/r^2$ also have finite energy and cannot be ignored. It turns out that these modes have energy which is even more negative than the $1/r^2$ modes. The easiest way to show that naked singularities can be produced is to modify our initial data. We consider the following class of configurations,

$$\phi(r) = \phi_0 = \frac{A}{\ln R_1} \frac{\ln R_0}{R_0^2} \quad (r \leq R_0)$$

$$\phi(r) = \frac{A}{\ln R_1} \frac{\ln r}{r^2} \quad (R_0 < r < R_1) \quad (3.3)$$

Since $\phi(R_1) \neq 0$, if a black hole forms, it must have some scalar hair. If one compares the mass of a static black hole with hair to our initial data, one finds that the black hole has larger mass. Since the energy is conserved in this regulated theory, the singularity must be naked. Since we can perturb the initial data without changing the conclusion, this yields generic violation of cosmic censorship in supergravity.

This suggests a new approach to studying cosmological singularities in string theory. We have argued that our initial data will form a singularity which is spacelike for a while in AdS. But our initial data is time symmetric, so there is a singularity in the past as well as the future. The solution thus looks like a homogeneous cosmology with a big bang and big crunch singularity embedded inside an asymptotically AdS spacetime (see Fig. 3). What is the dual CFT description? In this case, the CFT is $\mathcal{N} = 4$ super Yang-Mills. The operators dual to the fields which saturate the BF bound in AdS correspond to the operators $Tr[X^i X^j - (1/6)\delta^{ij} X^2]$ where X^i are the six scalars in the super Yang-Mills theory. (For a detailed discussion of the relation between the bulk fields and the boundary operators see [22, 23].) The large radius cut-off in AdS corresponds to a short distance cut-off in the gauge theory. It should be possible to map our initial data into the gauge theory and study its evolution. There appears to be no reason for this evolution to stop. This implies a string theory resolution of naked singularities. In principle, one should be able to use this to determine if universes can bounce through a cosmological singularity. The idea is to reconstruct the bulk description of the evolved state in the gauge theory. If the state corresponds to a semiclassical metric which is well defined everywhere shortly after the spacelike singularity, it would show that universes can bounce in string theory (as often assumed [24]). However, if it is only well defined outside a finite region, then there is no bounce. The naked singularity continues in the bulk and the cosmological singularity in the central region is truly an end of space and time.

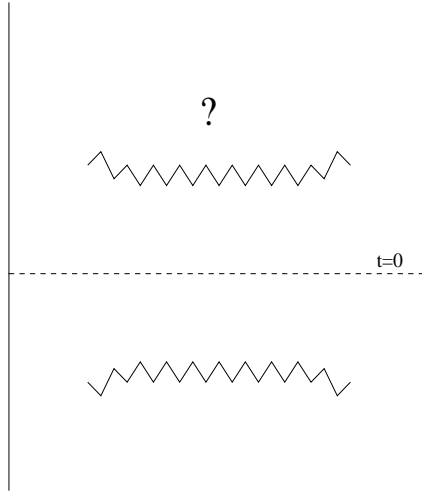


Figure 3: *Our solutions are like homogeneous universes beginning in a Big Bang singularity and ending in a Big Crunch, embedded in an asymptotically anti de Sitter space. The dual field description can be used to study how the singularities are resolved.*

4 Violation of cosmic censorship in asymptotically flat spacetimes?

Suppose we add a constant to the potential in Fig. 1 so that the local minimum at $\phi = \phi_1$ is at $V = 0$. Then there are asymptotically flat solutions and it is natural to ask if this theory also violates cosmic censorship. We still have the result that singularities are easier to form since nearly homogeneous regions of ϕ rolling down the negative part of the potential will still produce curvature singularities. But now black holes are not harder to form since $M_{bh} \propto R_s$. So the outcome depends on more details of the evolution. It should be straightforward to test whether cosmic censorship is violated in this theory using numerical relativity, since one can start with spherically symmetric configurations.

It is natural to ask why we should consider potentials with $V < 0$. After all, they violate the dominate energy condition. The answer is that they arise in many supersymmetric compactifications in string theory [3].

String theorists often consider spacetimes that approach $M_4 \times K$ asymptotically where M_4 is four dimensional Minkowski spacetime and K is a compact Ricci flat space admitting a covariantly constant spinor. Examples of K include T^n , four di-

mensional K3, six dimensional Calabi-Yau spaces, and seven dimensional manifolds with G_2 holonomy. Let us ignore the other fields in string theory and just consider vacuum solutions to general relativity in higher dimensions compactified down to four dimensions. In other words, we are just studying Kaluza-Klein theory. Physically one is often interested in the four dimensional effective description of these solutions. In four dimensions one has various matter fields coming from the metric with various interactions. We claim that some compactifications have configurations with negative four dimensional energy density, and in fact, this energy density can be arbitrarily negative.

To justify this claim, let's begin with the following mathematical question: Which compact manifolds K admit Riemannian metrics of positive scalar curvature? You might think that the answer is all of them since $\mathcal{R} > 0$ is one scalar inequality on the entire metric. Indeed, if you change the sign, this would be correct: All K with dimension greater than two admit metrics with $\mathcal{R} < 0$. But there is a topological obstruction to finding metrics with $\mathcal{R} > 0$ [25]. If you look at the standard examples above, T^n and K3 do not admit any such metrics, while all simply connected Calabi-Yau and G_2 manifolds do. There is a theorem proved in 1990 [26] which shows that in dimensions greater than four, all simply connected manifolds of dimension $3, 5, 6, 7 \bmod 8$ admit metrics with $\mathcal{R} > 0$. Note that this includes the cases of most interest to string theory: six and seven.

What does this have to do with negative energy density? Consider vacuum solutions in higher dimension that approach $M_4 \times K$ asymptotically. We can characterize this solution in terms of its initial data on a spacelike surface $\mathbf{R}^3 \times K$. For time symmetric initial data, the constraint equations reduce to $\mathcal{R} = 0$. If we take a product metric on $\mathbf{R}^3 \times K$ with $\mathcal{R}_K > 0$, then the only way to satisfy the constraint is to take $\mathcal{R}_3 < 0$. But the usual $3 + 1$ constraint is $\mathcal{R}_3 = 16\pi\rho$, so negative scalar curvature is just like negative energy density. Since we can make \mathcal{R}_K arbitrarily large by scaling K , we can make the energy density arbitrarily negative.

Of course we cannot keep the metric a product everywhere since we need the metric on K to approach the standard Ricci flat metric at infinity. But we can start with a large ball in \mathbf{R}^3 and put on a product metric with $\mathcal{R}_K > 0$. Then we can have a transition region where the metric on K approaches the Ricci flat one, and the metric on \mathbf{R}^3 is adjusted to still satisfy the constraint. This shows that Calabi-Yau

and G_2 compactifications have solutions with negative energy density. In other words, an effective four dimensional description would look qualitatively like Fig. 1 with a constant added so $V(\phi_1) = 0$. (This is only qualitative since a complete description would require an infinite number of fields in four dimensions.)

Despite having arbitrarily large regions of arbitrarily negative energy density, in this case the total ADM energy always remains positive. This has been shown recently using a Witten style positive energy theorem [27].

5 Conclusion

One can summarize the above discussion with two slogans: Supersymmetry does not always imply positive energy, and positive energy does not imply cosmic censorship.

There are many open questions remaining. As we have just discussed, one is whether cosmic censorship can be violated in asymptotically flat spacetimes using potentials which are not positive definite. Another is how common is cosmic censorship violation in supergravity. Are there examples which do not involve negative energies? Our examples of naked singularities have all begun with (nearly) time symmetry initial data, so there is a singularity in the past as well as the future. While this may be desirable for modeling the big bang, it would be of interest to show that naked singularities could be produced in an evolution with no singularities in the past. This seems likely, but has not been established.

I would like to conclude by pointing out that if cosmic censorship is violated in nature, it would not be a disaster. To the contrary, it would open up the possibility of directly observing the effects of Planck scale or string scale curvature. This would be exciting development for both observational astrophysicists and quantum gravity theorists.

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References

- [1] T. Hertog, G. T. Horowitz and K. Maeda, “Generic cosmic censorship violation in anti de Sitter space,” arXiv:gr-qc/0307102.
- [2] T. Hertog, G. T. Horowitz and K. Maeda, “Negative energy in string theory and cosmic censorship violation,” arXiv:hep-th/0310054.
- [3] T. Hertog, G. T. Horowitz and K. Maeda, “Negative energy density in Calabi-Yau compactifications,” JHEP **0305** (2003) 060 [arXiv:hep-th/0304199].
- [4] S. W. Hawking and R. Penrose, “The Singularities Of Gravitational Collapse And Cosmology,” Proc. Roy. Soc. Lond. A **314** (1970) 529.
- [5] R. Penrose, “Gravitational Collapse: The Role Of General Relativity,” Riv. Nuovo Cim. **1** (1969) 252 [Gen. Rel. Grav. **34** (2002) 1141].
- [6] R. M. Wald, “Gravitational collapse and cosmic censorship,” arXiv:gr-qc/9710068; P. S. Joshi, “Cosmic Censorship: A Current Perspective,” arXiv:gr-qc/0206087.
- [7] D. M. Eardley and L. Smarr, “Time Function In Numerical Relativity. Marginally Bound Dust Collapse,” Phys. Rev. D **19** (1979) 2239; D. Christodoulou, “Violation Of Cosmic Censorship In The Gravitational Collapse Of A Dust Cloud,” Commun. Math. Phys. **93** (1984) 171; P. S. Joshi and I. H. Dwivedi, “Naked Singularities In Spherically Symmetric Inhomogeneous Tolman-Bondi Dust Cloud Collapse,” Phys. Rev. D **47** (1993) 5357 [arXiv:gr-qc/9303037].
- [8] M. W. Choptuik, “Universality And Scaling In Gravitational Collapse Of A Massless Scalar Field,” Phys. Rev. Lett. **70** (1993) 9.
- [9] K. Maeda, T. Torii and M. Narita, “Do naked singularities generically occur in generalized theories of gravity?,” Phys. Rev. Lett. **81** (1998) 5270 [arXiv:gr-qc/9810081].
- [10] S. J. Poletti, J. Twamley and D. L. Wiltshire, “Charged dilaton black holes with a cosmological constant,” Phys. Rev. D **51** (1995) 5720 [arXiv:hep-th/9412076].

- [11] J. M. Maldacena, “The large N limit of superconformal field theories and supergravity,” *Adv. Theor. Math. Phys.* **2** (1998) 231 [*Int. J. Theor. Phys.* **38** (1999) 1113] [arXiv:hep-th/9711200].
- [12] O. Aharony, S. S. Gubser, J. M. Maldacena, H. Ooguri and Y. Oz, “Large N field theories, string theory and gravity,” *Phys. Rept.* **323** (2000) 183 [arXiv:hep-th/9905111].
- [13] L. F. Abbott, S. Deser, “Stability of Gravity with a Cosmological Constant,” *Nucl. Phys.* **B195** (1982) 76; A. Ashtekar and A. Magnon, “Asymptotically anti-de Sitter space-times”, *Class. Quantum Grav.* **1** (1984) L39.
- [14] G. W. Gibbons, C. M. Hull and N. P. Warner, “The Stability Of Gauged Supergravity,” *Nucl. Phys. B* **218** (1983) 173.
- [15] W. Boucher, “Positive Energy Without Supersymmetry,” *Nucl. Phys. B* **242** (1984) 282.
- [16] P. K. Townsend, “Positive Energy And The Scalar Potential In Higher Dimensional (Super)Gravity Theories,” *Phys. Lett. B* **148** (1984) 55.
- [17] D. Garfinkle, work in progress.
- [18] P. Breitenlohner and D. Z. Freedman, “Stability In Gauged Extended Supergravity,” *Annals Phys.* **144** (1982) 249; “Positive Energy In Anti-De Sitter Backgrounds And Gauged Extended Supergravity,” *Phys. Lett. B* **115** (1982) 197.
- [19] L. Mezincescu, P. K. Townsend, “Stability at a Local Maximum in Higher Dimensional Anti-de Sitter Space and Applications to Supergravity,” *Ann. Phys.* **160** (1985) 406.
- [20] E. Witten, “A Simple Proof Of The Positive Energy Theorem,” *Commun. Math. Phys.* **80** (1981) 381.
- [21] M. Henneaux, C. Teitelboim, “Asymptotically Anti-de Sitter Spaces,” *Comm. Math. Phys.* **98** (1985) 391.

- [22] I.R. Klebanov, E. Witten, “AdS/CFT Correspondence and Symmetry Breaking,” Nucl. Phys. **B556** (1999) 89 [arXiv:hep-th/9905104].
- [23] M. Bianchi, D. Z. Freedman and K. Skenderis, “How to go with an RG flow,” JHEP **0108** (2001) 041 [arXiv:hep-th/0105276]; “Holographic renormalization,” Nucl. Phys. B **631** (2002) 159 [arXiv:hep-th/0112119].
- [24] M. Gasperini and G. Veneziano, “The pre-big bang scenario in string cosmology,” Phys. Rept. **373** (2003) 1 [arXiv:hep-th/0207130]; P. J. Steinhardt and N. Turok, “Cosmic evolution in a cyclic universe,” Phys. Rev. D **65** (2002) 126003 [arXiv:hep-th/0111098].
- [25] B. Lawson and M. Michelsohn, *Spin Geometry*, Princeton University Press, Princeton, NJ (1989).
- [26] S. Stolz, “Simply Connected Manifolds of Positive Scalar Curvature”, Bull. Amer. Math. Soc. **23**, 427 (1990).
- [27] X. Dai, “A Positive Mass Theorem for Spaces with Asymptotic SUSY Compactification”, math.DG/0308249.